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Reply by Authors to R. H. Smith

K. Washizu* and K. Miyajima†
University of Tokyo, Tokyo, Japan

THE objective of the comment by R. H. Smith seems to point out that the controllability limits as given by Refs. 1 and 2 disagree widely because of the difference of practice, and that predictions of the limits of manual control made by Ref. 1 are not capable of yielding the limits of Ref. 2. It is noted at the beginning that we shall confine subsequent discussions to second-order controlled elements with positive static stability, namely, with positive stiffness, unless otherwise stated.

Replying to the first part of the comment, we agree that practice is an essential factor for explaining the discrepancy, although it should be remembered that no system input is present in the experiment of Ref. 1, whereas random inputs are present in that of Ref. 2. As mentioned in Ref. 1, we conducted experiments for obtaining the difference between the controllability limits at one trial and at three trials and concluded that the results indicated, evidently, the effect of practicing.

The controllability limit of a human operator may depend on many factors such as practice, his naiveté, or experience as an airplane pilot, and so forth. Consequently, it is considered indispensable when presenting data on the controllability limit to prescribe conditions fully under which experiments have been conducted. We believe that our data show *a* controllability limit under the prescribed test program of Ref. 1, namely, a controllability limit at one trial and at three trials. We note that our data agree very well with those obtained by NASA in Ref. 3.

On the other hand, we find in Ref. 2 that the two controllability limits, the establishment of which required nearly 900 trial runs, show clearly the effect of heavy practice. This is *a* controllability limit established by Smith. It seems probable that a more sophisticated technique would further improve the controllability limit established by him. We note here that unpublished data obtained recently at National Aerospace Laboratory of Japan also show that the controllability limit can be improved by heavy practice far beyond that of Ref. 1.

In replying to the second part of the comment, we agree that a transfer function, which is more complicated than Eq. (3) of Ref. 1 must be employed in predicting the limits of Ref. 2. The purpose of writing Ref. 1 was this: we assumed that the transfer function of a human pilot was given by Eq. (3) and that the pilot could conduct a self-adaptive control. Then, we made a speculation to find how our prediction could correlate with our experimental data. During the prediction, we were satisfied with obtaining a rough idea of the controllability limit by taking $T_L = \infty$, although it was possible to take an assigned value for T_L. We believe that Eq. (3) is an appropriate transfer function of a human pilot who is trying to find a controllability limit on a "first-encounter basis."

Received July 30, 1965.

* Professor of Aeronautics. Member AIAA.

† Graduate Student.

A brief mention is made here on an operator's behavior under heavy practice. Since there exists a periodicity in transient response of the controlled element with positive static stability even if the response may diverge, the operator is aware of the frequency and time to double amplitude of the response during a series of the practice. There exist indications that have led us to believe that, as the practice proceeds, the operator can improve the controllability limit by taking the response characteristics into consideration and employing a second-or-more higher-order lead equalization or the so-called "quasi-precognitive" technique. Consequently, we may conclude that the operator under heavy practice is employing a more sophisticated technique than that expressed by Eq. (3).

The second-order lead equalization technique may probably be favorably employed only within some region of the static stability and damping. For example, too small or too large values of positive static stability may be unfavorable for an operator who wants to employ the improved technique. On the other hand, this technique may not be so favorably applicable to controlled elements with negative static stability as it is to those with positive static stability, since the transient response in the former is aperiodically divergent.

The authors wish to thank Smith for the comment and agree with him in stating that the effect of practice on the controllability limit of a human operator is a very important subject and in hoping that more research will be conducted to clarify its mechanics in the future.

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Combustion Instability in Gas Rockets

C. T. Bowman,* I. Glassman,† and L. Crocco‡
Princeton University, Princeton, N. J.

ROCKET motors burning premixed gaseous propellants have been used by several investigators[1–3] to study the longitudinal mode of high-frequency combustion instability. The experimental results of these investigators differ in one important aspect. Zucrow and Osborn[2] and Tsuji and Takeno[3] have observed one region of unstable operation located around the stoichiometric mixture ratio. Pelmas et al.[1] have found two unstable regions, one on either side of stoichiometric.§ This apparent difference in the observations of these investigators has resulted in some uncertainty

Received July 16, 1965. Research sponsored by Air Force Office of Scientific Research, Office of Aerospace Research, U. S. Air Force, Contract No. AF49(638)-1268.

* Graduate Student, Department of Aerospace and Mechanical Sciences, Guggenheim Laboratories for the Aerospace Propulsion Sciences. Student Member AIAA.

† Professor, Department of Aerospace and Mechanical Sciences, Guggenheim Laboratories for the Aerospace Propulsion Sciences. Associate Fellow Member AIAA.

‡ Robert H. Goddard Professor, Department of Aerospace and Mechanical Sciences, Guggenheim Laboratories for the Aerospace Propulsion Sciences. Fellow Member AIAA.

§ An unstable region is a region in the equivalence ratio-combustion chamber length plane or equivalence ratio-combustion pressure plane where combustion pressure oscillations are observed.